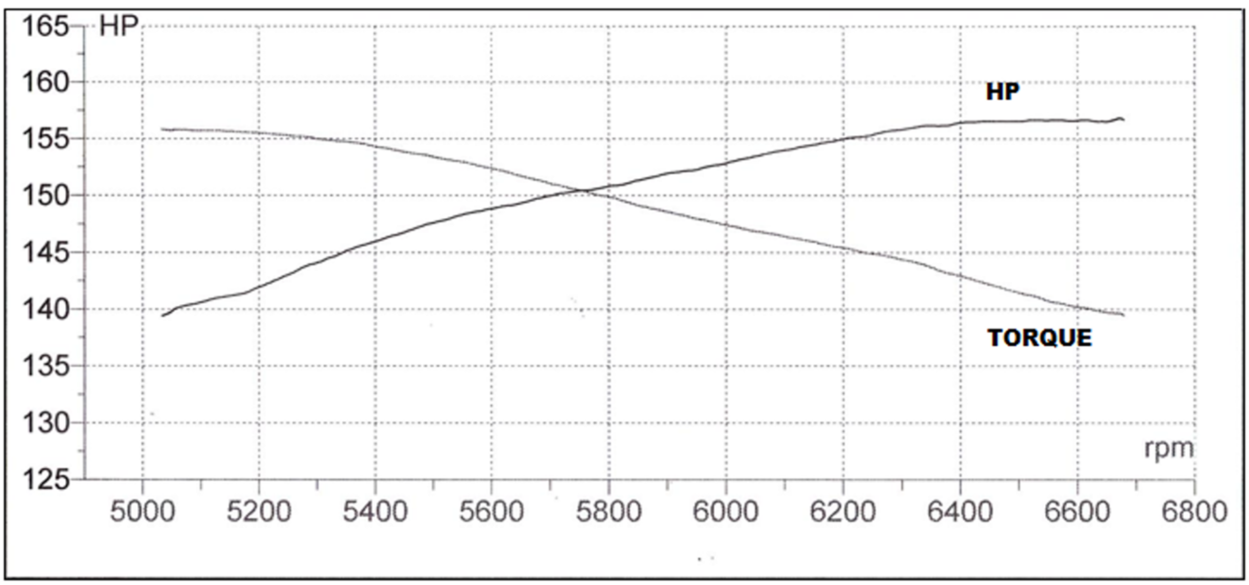
Numerical Analysis I - Honors Project

Nathan Murarik and Adam Sacherich

December 27, 2023

Introduction

This short paper discusses the Lagrange Interpolation, Newton's Divided Differences, and Cubic Spline Interpolation techniques to analyze a Torque curve and Horse Power curve recorded by a Road to Indy car. We will go over each of the techniques, and then analyze key properties of the curves as they pertain to the car itself. Below is a graph of the Road to Indy engine recording.



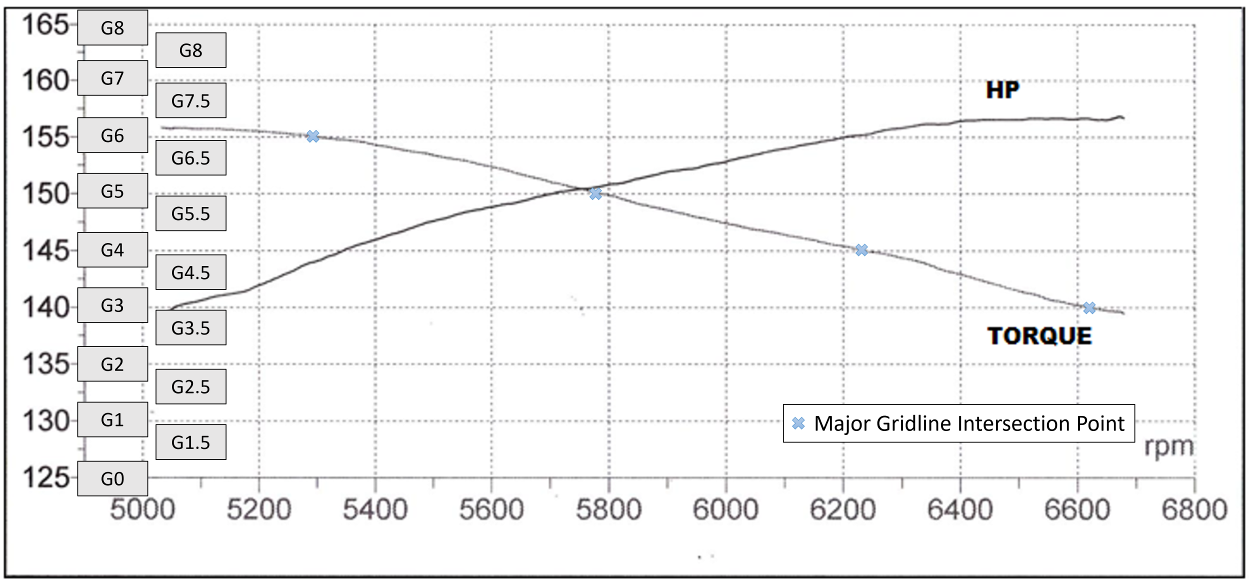
Research Methods

Torque Y-axis Axis Analysis using Power Equation

The provided plot lacked y-axis axis units for the torque data. To find the values corresponding to the major axes ticks on this axis, the spacing between grid lines first needed to be identified. First, each RPM value where the torque curve crossed a major or minor grid line was collected. These values were kept in order corresponding to what grid line they were associated with. The HP value for each of these RPMs was collected. Then, using the equation

$$T = \left(\frac{5252}{RPM}\right) \cdot \text{HP}$$

the expected torque value at each RPM was calculated. The following table outlines the collected data points with torque calculations and the following figure shows what torque value each grid line is associated with.



Grid Line ID	RPM Value	Plotted Power [HP]	Calculated Torque [ft.lb]
3	6616.237942	156.7757009	124.4492699
3.5	6425.241158	156.5514019	127.9653078
4	6235.691318	155.3551402	130.8475924
4.5	5985.369775	152.8130841	134.0893458
5	5777.009646	150.7196262	137.0223567
5.5	5580.948553	148.7757009	140.0066626
6	5293.729904	144.0654206	142.9297683

Grid line spacing could then be calculated by computing the difference in torque value between each data point. The resulting spacing values are summarized in the table below. Due to the limited resolution of original plot, there is some uncertainty when it comes to selecting an exact data point. In theory, all the grid lines should be equally spaced, but based on the calculated differences, the spacing varies. Since a uniform grid scaling is desired, the average grid line spacing of 6.1595194 was used.

The values of the grid lines were then calculated. By averaging the collected torque values, the theoretical torque value that lies on the minor tick between G4 and G5 can be calculated. Adding half of the grid spacing to this value results in finding that grid line G5 corresponds to a torque value of 136.7822127 ft*lb. This averaging method was used for the same reason described above. The torque values corresponding to all other grid lines could then be derived by adding multiples of the grid line spacing to G5. The resulting torque y-axis values are summarized in the table below.

Initial Grid Line ID	Final Grid Line ID	Absolute Difference in Torque [ft.lb]
3	3.5	2.923105698
3.5	4	2.984305907
4	4.5	2.933010894
4.5	5	3.241753395
5	5.5	2.882284618
5.5	6	3.516037942

Grid Line ID	Torque [ft.lb]
G0	106.181
G1	112.341
G2	118.501
G3	124.661
G4	130.821
G5	136.982
G6	143.142
G7	149.302
G8	155.462

Torque Y-axis Axis Analysis using Reported Values

While the method discussed in the previous section yielded promising results, it did not yield values accurate to the reported torque from the given information. The reason for why these calculated values do not agree with the provided data is unclear. This section focuses on a modified method to find the y-axis values based on the provided data.

Based on the reported data, the maximum torque value should be 134.7 ft*lb and the minimum should be 116 ft*lb. Using WebPlotDigitizer, the y-axis was defined by selecting the max of the torque curve and setting it to the given max torque and then doing the same for the minimum torque. The torque values of the major grid lines could then be extracted.

Grid Line ID	RPM Value	Torque [ft.lb]
3	6623.167421	116.595
4	6232.217195	122.29
5	5780.452489	128.07
6	5293.936652	133.765

The spacing between these grid lines was then calculated. These differences were averaged and the grid spacing was found to be 5.723 ft*lb.

By averaging the collected major axis values, the value of the G4.5 minor axis was found. Subtracting half of the major grid spacing yields the the value of the G4 major grid line. By adding integer multiples of the grid spacing to this value, the value of all other grid lines could be determined.

Initial Grid Line ID	Final Grid Line ID	Absolute Difference in Torque [ft.lb]
3	4	5.695
4	5	5.78
5	6	5.695

Grid Line ID	Torque [ft.lb]
G0	99.425
G1	105.148
G2	110.872
G3	116.595
G4	122.318
G5	128.042
G6	133.765
G7	139.488
G8	145.212

Data Point Capturing Method

All data points were collected using WebPlotDigitizer. This process was done by uploading the original plot into the software and then selecting known X and Y axis values. Any point in the plot could then be captured. Data points were selected approximately 100 RPM apart, and extra data points were collected near the end of the plots to estimate the derivatives.

Image Generation and Modification

All plots were generated using MATLAB and images were modified using Microsoft PowerPoint. To create superimposed plots, the original image plot was provided in PowerPoint along with the generated plot. The generated plot was turned to 50% opacity and then the grid lines of each plot were aligned. This process requires that the generated plots use the same x and y axis scaling as the original plot.

Lagrange Interpolation

Lagrange Interpolation interpolates a dataset at O(nlogn) as shown in Stoss's analysis of Lagrange Interpolation (Stoss, 1985). Thus, it is computationally viable to use while being more simple to implement than the following techniques. The biggest drawback to Lagrange's technique is it's risk of exponential decay or growth at the ends of the interpolating interval. The reader will notice, as a consequence, that our interpolation points are clustered towards the ends of the interpolating interval, but this issue cannot be entirely avoided when using Lagrange Interpolation.

Regardless, we start off with the expression

$$f(x) \approx p(x) = \sum_{k=0}^{n} f(x_k) \prod_{i=0, i\neq k}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

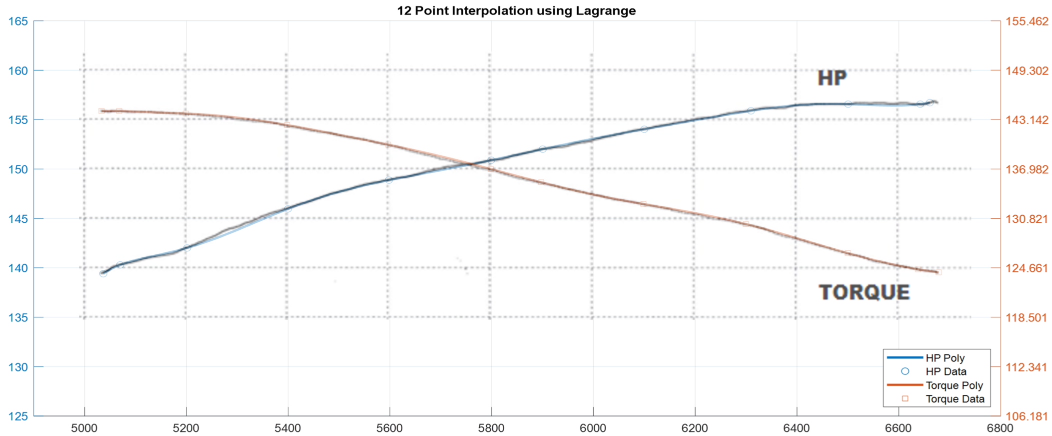
of a Lagrange Interpolation to interpolate the torque and horsepower nodes displayed below. We have also included the interpolation horsepower and torque equations and the graph of our interpolating polynomials.

n=	Torque Nodes	HP Nodes
0	(5034.751131, 134.7000)	(5036.665168, 139.4029851)
1	(5060.81448, 134.7000)	(5069.950548, 140.2985075)
2	(5199.819005, 134.2750)	(5201.578853, 142.0149254)
3	(5398.190045, 132.9150)	(5399.794102, 145.9701493)
4	(5598.00905, 130.7050)	(5597.971588, 148.8805970)
5	(5798.552036, 127.9000)	(5799.008272, 150.8955224)
6	(5909.321267, 126.2000)	(5900.253552, 152.0149254)
7	(6099.728507, 123.9050)	(6099.844452, 154.0298507)
8	(6299.547511, 121.6100)	(6309.550441, 155.8955224)
9	(6501.538462, 118.2100)	(6500.418090, 156.5671642)
10	(6636.199095, 116.4250)	(6642.104837, 156.5671642)
11	(6678.914027, 116.0000)	(6660.905413, 156.7164179)

$$\begin{aligned} HP(x) = &(-2.6508836 \cdot 10^{-31})x^{11} + (1.6916764 \cdot 10^{-26})x^{10} - (4.8994321 \cdot 10^{-22})x^{9} \\ &+ (8.5004015 \cdot 10^{-18})x^{8} - (9.8162341 \cdot 10^{-14})x^{7} + (7.9220943 \cdot 10^{-10})x^{6} - (0.0000045591799)x^{5} \\ &+ (0.018709948)x^{4} - (53.655047)x^{3} + (102399.46)x^{2} - (117047430.0)x + 6.0703551 \cdot 10^{6} \end{aligned}$$

$$\begin{aligned} T_{calculated}(x) = &(4.0544021 \cdot 10^{-31})x^{11} - (2.6209116 \cdot 10^{-26})x^{10} + (7.6922626 \cdot 10^{-22})x^{9} - (1.3530213 \cdot 10^{-17})x^{8} \\ &+ (1.5847485 \cdot 10^{-13})x^{7} - (1.297806 \cdot 10^{-9})x^{6} + (0.0000075827411)x^{5} - (0.031608902)x^{4} + (92.126658)x^{3} \\ &- (178798.89)x^{2} + 207965270.0x - 1.0982216 \cdot 10^{11} \end{aligned}$$

$$\begin{aligned} T_{actual}(x) = &(3.6481638 \cdot 10^{-31})x^{11} - (2.3576062 \cdot 10^{-26})x^{10} + (6.9173254 \cdot 10^{-22})x^{9} - (1.2163213 \cdot 10^{-17})x^{8} \\ &+ (1.4241594 \cdot 10^{-13})x^{7} - (1.1658913 \cdot 10^{-9})x^{6} + (0.0000068095855)x^{5} - (0.028375705)x^{4} + (82.672794)x^{3} \\ &- (160390.99)x^{2} + 186484270.0x - 9.8441125 \cdot 10^{10} \end{aligned}$$



Netwon's Divided Differences Interpolation

In general, it has been shown that Newton's Divided Differences generates a computationally, in floating-point arithmetic, more accurate polynomial than the Lagrange Interpolating Polynomial (Srivastava and Kumar Srivastava, 2012) even though their techniques generate, in theory, the same polynomial. Hence, we find it important to note the Netwon's Divided Differences formula is

$$f(x) \approx P(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, \ldots, x_k](x - x_0)\ldots(x - x_{k-1}) = \sum_{k=1}^{n} f[x_0, \ldots, x_k] \prod_{i=0}^{k-1} (x - x_i)$$

where $f[x_0] = f(x_0)$, $f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$, and $f[x_0, \ldots, x_k] = \frac{f[x_1, \cdots, x_k] - f[x_0, \ldots, x_{k-1}]}{x_k - x_0}$. We complete this section by showing an example of the horsepower divided differences for the first five terms using the data points outlined in the previous section.

Example (Newton's Divided Difference Applied to Horsepower Curve).

x_0	$f[x_0] = 139.402985$			
		$f[x_0, x_1] \approx 0.026904$		
x_1	$f[x_1] = 140.298507$		$f[x_0, x_1, x_2] \approx -8.40742 \cdot 10^{-5}$	
		$f[x_1, x_2] \approx 0.013039$		$[x_0, x_1, x_2, x_3] \approx 4.331480 \cdot 10^{-7}$
x_2	$f[x_2] = 142.014925$		$f[x_1, x_2, x_3] \approx 4.722541 \cdot 10^{-5}$	
		$f[x_2, x_3] \approx 0.028616$		$[x_1, x_2, x_3, x_4] \approx -1.559925 \cdot 10^{-7}$
x_3	$f[x_3] = 145.970149$		$f[x_2, x_3, x_4] \approx -3.514190 \cdot 10^{-5}$	
		$f[x_3, x_4] \approx 0.014686$		
x_4	$f[x_4] = 148.88059$			

where the last entry is $f[x_0, x_1, x_2, x_3, x_4] \approx -1.04958 \cdot 10^{-9}$. Therefore, we construct our polynomial as

$$\begin{aligned} HP(x) \approx\ & 139.402985 + 0.026904(x - 5036.665168) - (8.0472 \cdot 10^{-5})(x - 5036.665168)(x - 5069.950548) \\ & + (4.331480 \cdot 10^{-7})(x - 5036.665168)(x - 5069.950548)(x - 5201.578853) \\ & - (1.04958 \cdot 10^{-9})(x - 5036.665168)(x - 5069.950548)(x - 5201.578853)(x - 5399.794102) + \ldots \end{aligned}$$

Cubic Splines Interpolation

The Cubic Splines Interpolation method has an O(n) time complexity (Revesz, 2014), so it is worthwhile to construct a splines interpolation of our horsepower and torque curve. We also know that Runge's Phenomenon is mitigated by piecewise polynomials since their functions are defined on intervals and a splined together: rather than constructing an algebraic interpolation.

In lieu of the Cubic Splines method, we utilize the boundary conditions $f'(x_0) = S'(x_0) = \frac{f(x_{21})-f(x_0)}{x_{21}-x_0}$ and $f'(x_n) = S'(x_n) = \frac{f(x_{21})-f(x_0)}{x_{21}-x_0}$ (referred to as clamped splines) to interpolate our curve. This specific boundary condition is founded on our assumption that the Road to Indy engine is already generating horsepower before the recording of its horsepower curve (and consequentially reducing torque before the recording of its torque curve). Note that for the beginning of our Torque curve, we assume $T'(x_0) = 0$ because our interpretation of the graph appears to begin with a horizontal tangent.

We also assume the end of the horsepower curve and torque curve has a linear derivative since the system could still increase horsepower production (and thus decrease torque) in its testing run. Thus, with our assumptions, nodes, and clamped boundary conditions we generate the following table to represent the splines of the torque and horsepower curve where each spline is of the form $S_i = a_i+b_i(x-x_i)+c_i(x-x_i)^2+d_i(x-x_i)^3$. The x_i notation denotes the RPM of the ith element of the nodes table.

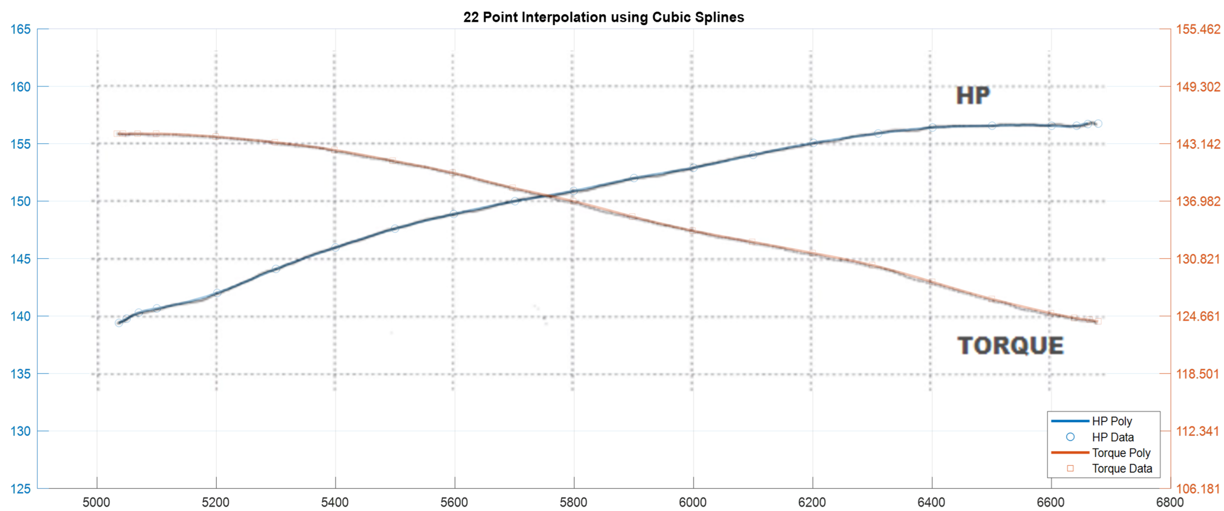
n=	TORQUE NODES	HP NODES
0	(5034.751131, 134.7000)	(5036.665168, 139.4029851)
1	(5060.81448, 134.7000)	(5049.690703, 139.7761194)
2	(5077.466063, 134.6575)	(5069.950548, 140.2985075)
3	(5099.909502, 134.6150)	(5100.325481, 140.6716418)
4	(5199.819005, 134.2750)	(5201.578853, 142.0149254)
5	(5301.176471, 133.6800)	(5299.967632, 144.1044776)
6	(5398.190045, 132.9150)	(5399.794102, 145.9701493)
7	(5499.547511, 131.8950)	(5499.612480, 147.6119403)
8	(5598.00905, 130.7050)	(5597.971588, 148.8805970)
9	(5700.81448, 129.1750)	(5700.662651, 150.0000000)
10	(5798.552036, 127.9000)	(5799.008272, 150.8955224)
11	(5909.321267, 126.2000)	(5900.253552, 152.0149254)
12	(5999.095023, 125.0525)	(6000.044956, 152.9104478)
13	(6099.728507, 123.9050)	(6099.844452, 154.0298507)
14	(6200.361991, 122.7575)	(6201.087035, 155.0746269)
15	(6299.547511, 121.6100)	(6309.550441, 155.8955224)
16	(6400.180995, 119.9100)	(6400.653659, 156.4179104)
17	(6501.538462, 118.2100)	(6500.418090, 156.5671642)
18	(6601.447964, 116.8075)	(6600.177126, 156.5671642)
19	(6636.199095, 116.4250)	(6642.104837, 156.5671642)
20	(6665.158371, 116.1700)	(6678.256159, 156.7537313)
21	(6678.914027, 116.0000)	(6660.905413, 156.7164179)

$$HP(X) \approx \begin{cases} -0.000014526143\,x^3 + 0.28984279\,x^2 - 1927.7485\,x + 4273954.3 & \text{if } x \in (6642.1048, 6678.2562) \\ 0.0000035999315\,x^3 - 0.071343074\,x^2 + 471.28585\,x - 1037591.6 & \text{if } x \in (6600.1771, 6642.1048) \\ -0.00000025418186\,x^3 + 0.0049704175\,x^2 - 32.396711\,x + 70539.741 & \text{if } x \in (6500.4181, 6600.1771) \\ 0.00000016784102\,x^3 - 0.003259558\,x^2 + 21.101571\,x - 45380.659 & \text{if } x \in (6400.6537, 6500.4181) \\ -0.00000013166419\,x^3 + 0.0024915295\,x^2 - 15.709148\,x + 33156.896 & \text{if } x \in (6309.5504, 6400.6537) \\ 0.000000051229027\,x^3 - 0.00097039252\,x^2 + 6.1340231\,x - 12783.301 & \text{if } x \in (6201.087, 6309.5504) \\ -0.000000028084257\,x^3 + 0.0005050932\,x^2 - 3.0155923\,x + 6129.2193 & \text{if } x \in (6099.8445, 6201.087) \\ -0.00000011718991\,x^3 + 0.002135685\,x^2 - 12.961949\,x + 26352.961 & \text{if } x \in (6000.045, 6099.8445) \\ 0.00000018373149\,x^3 - 0.0032809408\,x^2 + 19.538049\,x - 38647.522 & \text{if } x \in (5900.2536, 6000.045) \\ -0.00000018227827\,x^3 + 0.0031977105\,x^2 - 18.687636\,x + 36532.89 & \text{if } x \in (5799.0083, 5900.2536) \\ 0.0000001542976\,x^3 - 0.0026577084\,x^2 + 15.267987\,x - 29103.422 & \text{if } x \in (5700.6627, 5799.0083) \\ -0.000000048833278\,x^3 + 0.00081623352\,x^2 - 4.535784\,x + 8528.1167 & \text{if } x \in (5597.9716, 5700.6627) \\ 0.00000007285293\,x^3 - 0.0012273543\,x^2 + 6.9041625\,x - 12818.715 & \text{if } x \in (5499.6125, 5597.9716) \\ -0.000000077532604\,x^3 + 0.0012538322\,x^2 - 6.7414016\,x + 12196.39 & \text{if } x \in (5399.7941, 5499.6125) \\ 0.00000010436126\,x^3 - 0.0016927361\,x^2 + 9.1694604\,x - 16442.07 & \text{if } x \in (5299.9676, 5399.7941) \\ -0.00000031323879\,x^3 + 0.0049470641\,x^2 - 26.021266\,x + 45727.833 & \text{if } x \in (5201.5789, 5299.9676) \\ 0.000000076717931\,x^3 - 0.0011381077\,x^2 + 5.6312354\,x - 9153.1601 & \text{if } x \in (5100.3255, 5201.5789) \\ 0.0000048883544\,x^3 - 0.074760844\,x^2 + 381.13115\,x - 647543.76 & \text{if } x \in (5069.9505, 5100.3255) \\ -0.0000066625311\,x^3 + 0.10092641\,x^2 - 509.59454\,x + 857767.98 & \text{if } x \in (5049.6907, 5069.9505) \\ -0.00000018245633\,x^3 + 0.0027592909\,x^2 - 13.880948\,x + 23367.873 & \text{if } x \in (5036.6652, 5049.6907) \end{cases}$$

$$T_{calc}(x) \approx \begin{cases} 0.0000042205465\,x^3 - 0.084634491\,x^2 + 565.70917\,x - 1260269.5 & \text{if } x \in (6663.9871, 6678.4566) \\ -0.0000057339015\,x^3 + 0.11437445\,x^2 - 760.48385\,x + 1685641.5 & \text{if } x \in (6637.9421, 6663.9871) \\ 0.0000017177936\,x^3 - 0.034017313\,x^2 + 224.53208\,x - 493851.38 & \text{if } x \in (6598.8746, 6637.9421) \\ -0.00000012590204\,x^3 + 0.0024816363\,x^2 - 16.319914\,x + 35932.64 & \text{if } x \in (6499.7588, 6598.8746) \\ 0.000000094780396\,x^3 - 0.0018215115\,x^2 + 11.649509\,x - 24665.527 & \text{if } x \in (6399.9196, 6499.7588) \\ 0.00000011714352\,x^3 - 0.002250878\,x^2 + 14.397421\,x - 30527.665 & \text{if } x \in (6299.3569, 6399.9196) \\ -0.00000013077337\,x^3 + 0.0024342729\,x^2 - 15.116018\,x + 31444.229 & \text{if } x \in (6199.5177, 6299.3569) \\ 0.000000020300387\,x^3 - 0.00037548043\,x^2 + 2.303098\,x - 4552.4763 & \text{if } x \in (6098.2315, 6199.5177) \\ -0.000000078843387\,x^3 + 0.0014383246\,x^2 - 8.7579051\,x + 17931.71 & \text{if } x \in (5998.3923, 6098.2315) \\ 0.000000012587312\,x^3 - 0.00020698697\,x^2 + 1.1113193\,x - 1801.4501 & \text{if } x \in (5897.1061, 5998.3923) \\ 0.00000015804603\,x^3 - 0.0027803435\,x^2 + 16.286676\,x - 31631.679 & \text{if } x \in (5797.9904, 5897.1061) \\ -0.00000020342643\,x^3 + 0.003507098\,x^2 - 20.167849\,x + 38822.649 & \text{if } x \in (5696.7042, 5797.9904) \\ 0.00000021157134\,x^3 - 0.0035852605\,x^2 + 20.235219\,x - 37898.793 & \text{if } x \in (5596.1415, 5696.7042) \\ -0.00000013390304\,x^3 + 0.0022147101\,x^2 - 12.222237\,x + 22646.711 & \text{if } x \in (5495.5788, 5596.1415) \\ 0.00000011376561\,x^3 - 0.0018685377\,x^2 + 10.217573\,x - 18459.869 & \text{if } x \in (5397.91, 5495.5788) \\ -0.000000061548224\,x^3 + 0.00097044713\,x^2 - 5.1070117\,x + 9113.7064 & \text{if } x \in (5298.0707, 5397.91) \\ 9.6478495e-9\,x^3 - 0.00016115837\,x^2 + 0.88831431\,x - 1474.1806 & \text{if } x \in (5198.955, 5298.0707) \\ 0.000000078369153\,x^3 - 0.0012329953\,x^2 + 6.460746\,x - 11131.121 & \text{if } x \in (5099.1158, 5198.955) \\ -0.00000047756718\,x^3 + 0.0072713559\,x^2 - 36.903925\,x + 62576.038 & \text{if } x \in (5068.0064, 5099.1158) \\ 0.00000019505776\,x^3 - 0.0029552467\,x^2 + 14.924563\,x - 24979.665 & \text{if } x \in (5043.4084, 5068.0064) \\ -0.00000019641705\,x^3 + 0.0029678553\,x^2 - 14.948059\,x + 25240.279 & \text{if } x \in (5033.2797, 5043.4084) \end{cases}$$

$$T_{actual}(x) = \begin{cases} 0.0000092929584\,x^3 - 0.18600395\,x^2 + 1240.9795\,x - 2759707.3 & \text{if } x \in (6665.1584, 6678.914) \\ -0.0000031824544\,x^3 + 0.063447858\,x^2 - 421.65632\,x + 934202.93 & \text{if } x \in (6636.1991, 6665.1584) \\ 0.00000069695147\,x^3 - 0.013785671\,x^2 + 90.880755\,x - 199563.09 & \text{if } x \in (6601.448, 6636.1991) \\ 9.9618786e-9\,x^3 - 0.00018029286\,x^2 + 1.0655592\,x - 1926.3084 & \text{if } x \in (6501.5385, 6601.448) \\ 0.000000017961099\,x^3 - 0.00033631457\,x^2 + 2.0799404\,x - 4124.6544 & \text{if } x \in (6400.181, 6501.5385) \\ 0.00000017680584\,x^3 - 0.0033862199\,x^2 + 21.599886\,x - 45768.383 & \text{if } x \in (6299.5475, 6400.181) \\ -0.0000001873898\,x^3 + 0.0034965834\,x^2 - 21.75866\,x + 45278.024 & \text{if } x \in (6200.362, 6299.5475) \\ 0.000000050054164\,x^3 - 0.00092013223\,x^2 + 5.6265758\,x - 11321.434 & \text{if } x \in (6099.7285, 6200.362) \\ -0.000000033055322\,x^3 + 0.00060070368\,x^2 - 3.6501103\,x + 7540.3213 & \text{if } x \in (5999.095, 6099.7285) \\ -0.000000075715103\,x^3 + 0.0013684639\,x^2 - 8.2559769\,x + 16750.665 & \text{if } x \in (5909.3213, 5999.095) \\ 0.00000016925906\,x^3 - 0.0029744292\,x^2 + 17.407573\,x - 33800.723 & \text{if } x \in (5798.552, 5909.3213) \\ -0.00000019766086\,x^3 + 0.0034083835\,x^2 - 19.603498\,x + 37736.151 & \text{if } x \in (5700.8145, 5798.552) \\ 0.0000001768956\,x^3 - 0.0029974471\,x^2 + 16.914954\,x - 31658.822 & \text{if } x \in (5598.009, 5700.8145) \\ -0.000000072116639\,x^3 + 0.0011844712\,x^2 - 6.4954625\,x + 12025.086 & \text{if } x \in (5499.5475, 5598.009) \\ 0.000000024653721\,x^3 - 0.00041210838\,x^2 + 2.2850027\,x - 4071.1091 & \text{if } x \in (5398.19, 5499.5475) \\ -0.000000012567863\,x^3 + 0.00019067917\,x^2 - 0.96895903\,x + 1784.0589 & \text{if } x \in (5301.1765, 5398.19) \\ 7.1089745e-9\,x^3 - 0.00012225199\,x^2 + 0.68994428\,x - 1147.3209 & \text{if } x \in (5199.819, 5301.1765) \\ 0.000000025516933\,x^3 - 0.00040940615\,x^2 + 2.1830939\,x - 3735.3568 & \text{if } x \in (5099.9095, 5199.819) \\ -0.000001113855\,x^3 + 0.017022676\,x^2 - 86.718946\,x + 147395.43 & \text{if } x \in (5077.4661, 5099.9095) \\ 0.0000034957987\,x^3 - 0.053193405\,x^2 + 269.80082\,x - 456010.24 & \text{if } x \in (5060.8145, 5077.4661) \\ -0.0000022759843\,x^3 + 0.034436364\,x^2 - 173.67718\,x + 292109.72 & \text{if } x \in (5034.7511, 5060.8145) \end{cases}$$

Lastly, the graph of our interpolation (with calculated torque) is shown below.



Horse Power and Torque Analysis

We divide this section into two subsections. The first subsection entails an analysis of the Lagrange/Netwon's Interpolating Polynomial, generated in previous sections, with the utilization of the Bisection Method and Secant Method for approximating its roots. The second subsection entails our piece-wise cubic spline polynomial with the same root-finding algorithms as with the Lagrange polynomial.

Lagrange/Newton's Interpolation Analysis

Before we delve into the root-finding methodologies themselves, note the $T'(x_0) \approx 0.002285$ and $T'(x_{21}) \approx -0.008668$ which implies by the Intermediate Value Theorem that a root of T's derivative exists on the interpolating interval. Thus, it appears that it's best for us to utilize the Bisection Method when analyzing min-max extrema on our interval, provided we can reduce our interval into subsets of the ambient interval to find all roots. For the Horse Power we have $HP'(x_0) \approx 0.035474...$ and $HP'(x_{21}) \approx 0.009432...$ so we can't make a conclusion. Thus, we hypothesize the secant method works more efficiency for our Horse Power curve. Below is a collection of our results from the Bisection Method and Secant Method with an error bound of $\epsilon = 10^{-20}$ to find accurate roots, regardless of whether they fulfill the ϵ-bounds or not. The code reference is attached as supplementary material.

Table 1: Bisection Method with Lagrange Interpolation

Property	(RPM, HP)	Derivative
Minimum Torque	(6678.914027, 116.000000)	$7.358108 \cdot 10^{-17}$
Maximum Torque	(5046.814473, 134.712895)	$3.532954 \cdot 10^{-11}$
Minimum Horsepower	(5036.66518, 139.402985)	$1.2784495 \cdot 10^{-11}$
Maximum Horsepower	(6646.777428, 156.5957501)	$1.2784495 \cdot 10^{-11}$

Table 2: Secant Method with Lagrange Interpolation

Property	(RPM, T)	Derivative
Minimum Torque	(6678.914027, 116.000000)	$7.358108 \cdot 10^{-17}$
Maximum Torque	(5046.814473, 134.712895)	$3.532954 \cdot 10^{-11}$
Minimum Horsepower	(5036.66518, 139.402985)	0.03547469
Maximum Horsepower	(6589.8296614, 156.41455667)	$-3.153830 \cdot 10^{-17}$

We use the two tables to estimate the minimum and maximum for the Torque and Horsepower curves. We have also included the calculated average of our curves, found via MATLAB, as shown below.

Table 3: Summary of Lagrange Interpolation Extrema, Range, and Average

Property	Value	Derivative
Minimum Torque	(6678.914027, 116.000000)	$7.358108 \cdot 10^{-17}$
Maximum Torque	(5046.814473, 134.712895)	$3.532954 \cdot 10^{-11}$
Minimum Horsepower	(5036.66518, 139.402985)	$1.2784495 \cdot 10^{-11}$
Maximum Horsepower	(6589.8296614, 156.41455667)	$-3.153830 \cdot 10^{-17}$
Torque Range	18.712895	n/a
Horsepower Range	17.011571	n/a
Average Torque	135.739811	n/a
Average Horsepower	150.527616	n/a

Cubic Splines Interpolation Analysis

Via the condition for our Cubic Splines for our Torque curve, we know $T'(x_0) = 0$ which implies x_0 yields an minimum or maximum. Consequentially, x_0 is a maximum since the rest of the torque curve is a decreasing function. We also have for HP that $HP'(x_0) = 0.028646$ and $HP'(x_{21}) = 0.005197$ so no conclusion can be made about the horse power curve. Below is a collection of our results from the Bisection Method and Secant Method with an error bound of $\epsilon = 10^{-20}$ to find accurate roots, regardless of whether they fulfill the ϵ-bounds or not. The code reference is attached as supplementary material.

Table 4: Bisection Method with Cubic Interpolation

Property	(RPM, HP)	Derivative
Minimum Torque	(6678.914027, 116.000000)	-0.008668
Maximum Torque	(5034.751131, 134.700000)	0
Minimum Horsepower	(5036.665168, 139.4029851)	0.03547469
Maximum Horsepower	(6560.891246, 156.443354)	-0.0017627028

Table 5: Secant Method with Cubic Interpolation

Property	(RPM, T)	Derivative
Minimum Torque	(6678.914027, 116.000000)	-0.008668
Maximum Torque	(5034.751131, 134.700000)	0
Minimum Horsepower	(5036.665168, 139.4029851)	0.03547469
Maximum Horsepower	(6560.891246, 156.443354)	-0.0017627028

We use the two tables to estimate the minimum and maximum for the Torque and Horsepower curves to be as shown below.

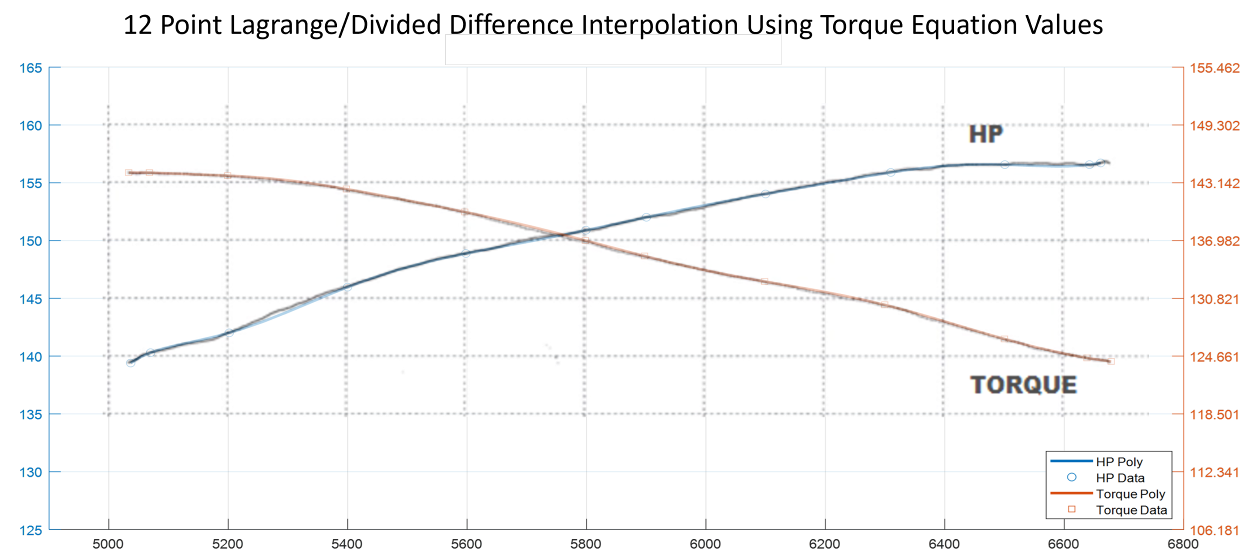
Table 6: Summary of Cubic Interpolation Extrema, Range, and Average

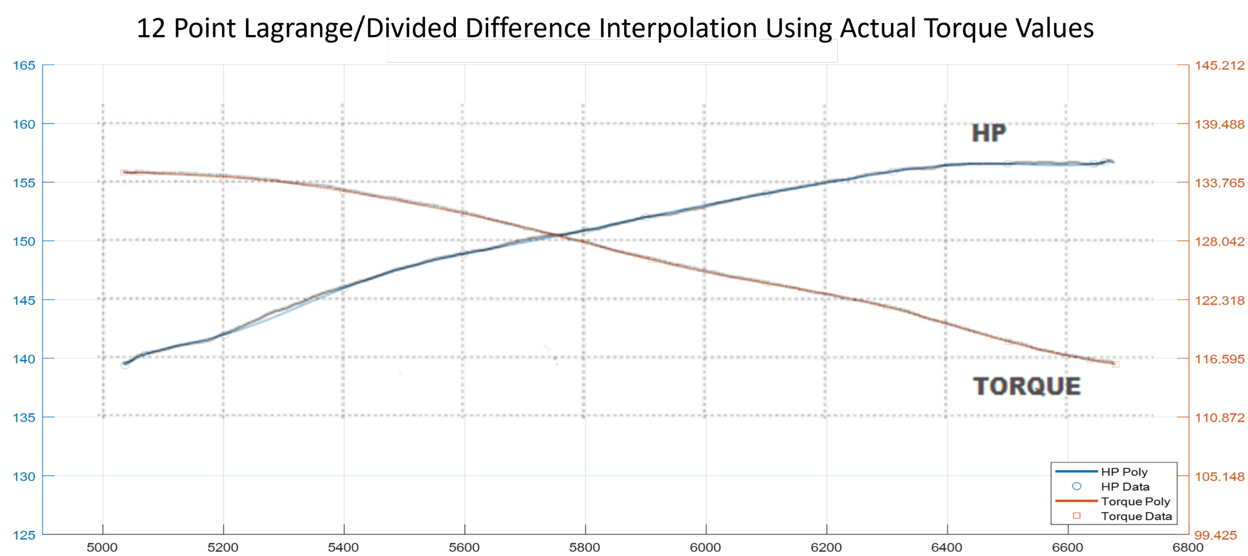
Property	Value	Derivative
Minimum Torque	(6678.914027, 116.000000)	-0.008668
Maximum Torque	(5034.751131, 134.700000)	0.002983155
Minimum Horsepower	(5036.665168, 139.4029851)	0.03547469
Maximum Horsepower	(6560.891246, 156.443354)	-0.0017627028
Torque Range	18.700000	n/a
Horsepower Range	17.040369	n/a
Average Torque	135.72161	n/a
Average Horsepower	150.542927	n/a

Conclusion

Below are the resulting outputs of the interpolation outputs. Lagrange and Divided differences are put together as the both result in the same polynomial.

Lagrange/Divided Differences

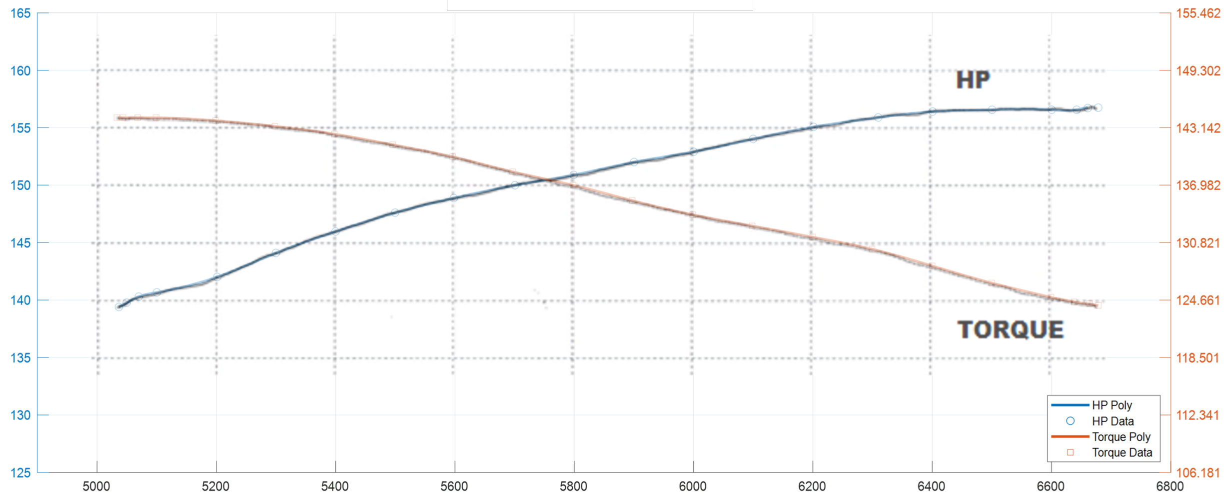




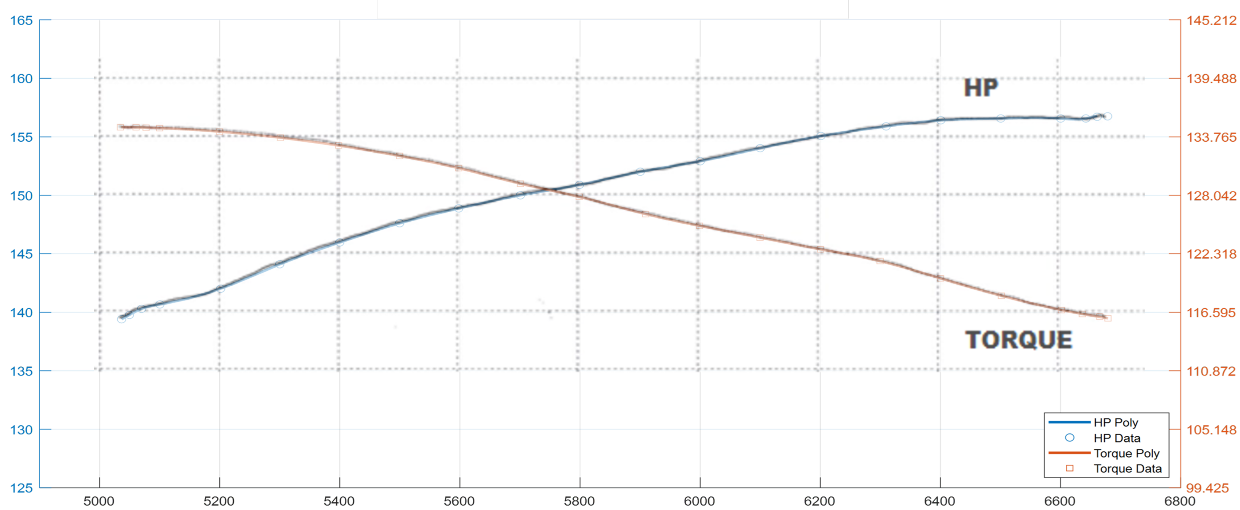
Average Power = 150.5276 HP
Average Torque (Equation Method) = 135.7398 ft*lb
Average Torque (Data Method) = 126.7586 ft*lb

Clamped Cubic Splines

22 Clamped Cubic Spline Interpolation Using Torque Equation Values



22 Clamped Cubic Spline Interpolation Using Actual Torque Values



Average Power = 150.5429 HP
Average Torque (Equation Method) = 135.7216 ft*lb
Average Torque (Data Method) = 126.7412 ft*lb